Chord Geometries 1.1

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ChordGeometries represents chords and voice leadings in a variety of geometrical spaces. You can enter chords on a MIDI keyboard or using the Keyboard window. Voice leadings between successive chords are represented by continuous paths in the spaces. For more information, see Dmitri Tymoczko's "The Geometry of Musical Chords."[1] Further information can be found in "Generalized Chord Spaces," a work-in-progress with Clifton Callender and Ian Quinn.[2]

I. Installation and System Requirements

ChordGeometries was made using Max/Jitter from Cycling 74.[3] The program runs as a freestanding application for Macintosh and Windows XP. The Macintosh application requires OS10.3 or higher, 256MB of RAM, QuickTime 6.0 or higher, and OpenGL 1.5 or higher. **NB:** Macintosh users *must* unzip the "ChordGeometries.zip" archive using the application BOMArchiveHelper. You can do this by control-clicking on the archive's icon, and choosing "Open With ..."

The Windows XP application requires 256 MB of RAM, a 1 GHz processor or higher, an OpenGL hardware-accelerated video card, QuickTime 6.0, and OpenGL 1.5 or later. **NB:** PC users may need to download the latest version of Quicktime.[4] In rare circumstances, users may also need to download the latest driver for their video card.[5]

II. Using the Program

A. Notation. Pitch classes are represented by integers. 0 = C , 1 = C♯/D♭, 2 = D, 3 = D♯/E♭, and so on. "T" and "E" correspond to 10 (A♯/B♭) and 11 (B), respectively.

B. Input. When using MIDI input, lift your hands off the keyboard to send a chord to the computer. When using the Keyboard window, press spacebar to send a chord. The "Auto-length" function in the Keyboard window will automatically send chords of the desired length to the computer. (Auto-length 0 requires the user to send every chord using the spacebar.) Selecting a geometry automatically sets this parameter.

The order in which notes are played tells the computer which voice has which note. For example, the ordered sequence C–E–G indicates that Voice 1 has C, Voice 2 has E, and Voice 3 has G. The ordered sequence E–C–G indicates that Voice 1 has E, Voice 2 has C, and Voice 3 has G. A pair of chords determines a continuous path in the geometries. For instance, in the "Circular Space" window entering the chord C–E followed by E–C will send one note along a path from C to E, and another note along a path from E to C.

C. Window manipulation. The 3D images can be rotated, scaled, and moved. To

[1] music.princeton.edu/~dmitri/voiceleading.pdf
[2] music.princeton.edu/~dmitri/chordspaces.pdf
[3] www.cycling74.com
[4] www.apple.com/quicktime/download/win.html
[5] See the manufacturer's website.

rotate, click and drag on the model. To move the image, command-click and drag (Macintosh) or control-click and drag (Windows XP). To zoom in or out, option-click and drag (Macintosh) or alt-click and drag (Windows XP).

D. Reset, Fullscreen, and Parameters. These commands are found in the "Commands" menu and affect the most recent geometry selected from the "Geometries" menu. "Reset" returns the parameters to their default state. "Fullscreen" toggles fullscreen mode. "Parameters" allows users to access the adjustable parameters specific to that geometry.

E. Play MIDI. Choose this command to play the notes entered using the Keyboard window.

F. Color. The color of a chord indicates how evenly it divides the octave. Red chords divide the octave perfectly evenly; blue chords divide it as unevenly as possible, and contain only a single pitch class. Chords that divide the octave nearly evenly are reddish or orange; clustered chords that divide the octave unevenly are bluish or green.

G. Performance. Your system will slow down when displaying complex 3D models. In case of emergency, you can stop the graphics by selecting "Stop or Restart Graphics" from the Commands menu (command-G on Macintosh, control-G on PC). To improve performance, you might try choosing a higher "redraw interval" in the "Master Parameters" window. You can also use the individual "Parameters" windows (section IV, below) to limit the chords displayed.

H. Chord Name. The "Chord Name" window attempts to name chords. If it does not know the chord name, it gives the name in a kind of atonal "figured bass" notation. Below the name it provides the chord's standard "normal form."

I. Chords in a scale. To show only the chords in a scale, open the "Master Parameters" window under "Commands." Then enter the pitch classes you wish to display, or choose one of the shortcuts from the pop-up menu.

III. Geometries.

A. Linear Space. Pitches appear as points on the real line $\mathbb{R}$. Distance on the line corresponds to log-frequency distance. This space is a continuous analogue to the standard piano keyboard.

B. Circular Space. All octave-related notes are represented by a single point, a pitch class. The result is a circular space (*pitch class space*) that mathematicians call $\mathbb{R}/12\mathbb{Z}$. One can invert the space by selecting "Invert Pitch-Class Circle" from the "Commands" menu. Inversion is animated as a three-dimensional rotation that preserves points 0 and 6.

C. Dyadic Space. Unordered pairs of pitch classes are represented as points on the Möbius strip $\mathbb{T}^2/\mathcal{S}_2$. The top and bottom edges act as mirrors; the left and right edges are glued together modulo a half twist.

D. Dyadic Set-Class Space. Points in this space represent *interval classes,* or unordered pairs of pitch classes modulo transposition. One can think of this space as a vertical "slice" of Dyadic Space modulo reflection around its center point.

E. Triadic Space. Unordered triples of pitch classes are represented as points on the orbifold $\mathbb{T}^3/\mathcal{S}_3$, which is bounded by a twisted triangular 2-torus The figure appears as a triangular prism whose top and bottom faces are identified modulo rotation by 120 degrees.

F. Triadic Set-Class Space. Points in this space represent *transpositional set classes* or unordered triples of pitch classes modulo transposition. One can think of this space as a triangular (horizontal) "slice" of

Triadic Space modulo 120-degree rotation around its center point. The result is a cone.

G. Tetrachordal Space. The orbifold $\mathbb{T}^4/\mathcal{S}_4$ is four dimensional; therefore we can only look at part of it at one time. By default, the program displays a single tetrahedral slice of the orbifold, analogous to a vertical slice of Dyadic Space or a triangular slice of Triadic Space. The parameters window allows the user to view the quotients of this tetrahedron by the reflection that cyclically permutes the vertices of the tetrahedron; one can also look at the quotient of this space by an additional reflection. These latter spaces are the tetrachordal analogues to the dyadic and triadic set-class spaces.

IV. Parameters

A. Master parameters. "Speed" sets the speed of motion in all geometries. "Redraw interval" sets the time, in milliseconds, between screen redraws. Smaller numbers require more processing power. "Display these PCs" allows you to limit yourself to notes belonging to a specific scale. "Input" and "output" provide control over MIDI input and output.

B. Linear and Circular Space. "Speed" sets the speed of motion in the Linear and Circular spaces.

C. Dyadic Space and Dyadic Set Class Space. "Speed" sets the speed of the motion in these two spaces. "Preset" provides easy access to some useful settings. The next menu determines whether to draw lines between dyads connected by single-step oblique voice leading, stepwise parallel voice leading, stepwise contrary voice leading, or any combination of the above. "Four-note mode" allows the user to display two dyads at once in these spaces; one can thereby display four-note chords in two dimensions. "Permutation" determines how a four-note sequence is to be grouped into two dyads: the permutation in this window is applied to the notes of a four-note sequence, which is then separated into the pairs {1, 2} and {3, 4}. Thus "1 3 4 2" indicates that the chord is to be separated into pairs consisting of the first and last notes and the second and third notes.

D. Trichordal Space and Trichordal Set Class Space. One can use the menu to choose which set classes are displayed, or one can enter them manually; a variety of other presets are also provided. When "autorotate" is set, the models will continue rotating on their own. "Speed" sets the speed. "Cone height" allows users to change the height of the Triadic Set-Class cone: 0 flattens the cone into a teardrop shape, while 1 displays it at its normal height. Finally, users can choose to draw lines between chords connected by various kinds of stepwise voice leading.

E. Tetrachordal Space. One can use the menu to choose which set classes are displayed, or one can enter them manually. "Speed" sets the speed. "Graph type" determines whether you see a slice of the 4D orbifold ("permutation region") or one of the quotients of this space. "Transpositional set-class space" displays the quotient of the permutation region by a vertex-permuting reflection; there is one representative of every transpositional set class in this space. "TI-set classes" displays the quotient of *this* space by a further reflection. There is one representative of every TI-set class in this space. A further menu ("parsimonious lines") allows users to choose whether to draw lines between set classes connected by single-semitone voice leading. The final menu determines whether the model rotates automatically.